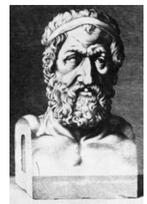
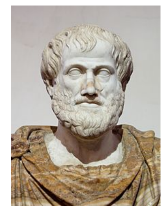
The Paradoxes of Motion: The Dichotomy*

The first asserts the non-existence of motion on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal. (Aristotle Physics, 239b11)

This paradox is known as the dichotomy because it involves repeated division into two (like the second paradox of plurality). Like the other paradoxes of motion we have it from Aristotle, who sought to refute it.

Suppose a very fast runnersuch as mythical Atalantaneeds to run for the bus. Clearly before she reaches the bus stop she must run half-way, as Aristotle says. There's no problem there; supposing a constant motion it will take her 1/2 the time to run half-way there and 1/2 the time to run the rest of the way. Now she must also run half-way to the half-way pointi.e., a 1/4 of the total distancebefore she reaches the half-way point, but again she is left with a finite number of finite lengths to run, and plenty of time to do it. And before she reaches 1/4 of the way she must reach 1/2 of 1/4 = 1/8 of the way; and before that a 1/16; and so on. There is no problem at any finite point in this series, but what if the halving is carried out infinitely many times? The resulting series contains no first distance to run, for any possible first distance could be divided in half, and hence would not be first after all. However it does contain a final distance, namely 1/2 of the way; and a penultimate distance, 1/4 of the way; and a third to last distance, 1/8 of the way; and so on. Thus the series of distances that Atalanta is required to run is: , then 1/16 of the way, then 1/8 of the way, then 1/4 of the way, and finally 1/2 of the way (of course we are not suggesting that she stops at the end of each segment and then starts running at the beginning of the nextwe are thinking of her continuous run being composed of such parts). And now there is a problem, for this description of her run has her travelling an infinite number of finite distances, which, Zeno would have us conclude, must take an infinite time, which is to say it is never completed. And since the argument does not depend on the distance or who or what the mover is, it follows that no finite distance can ever be traveled, which is to say that all motion is impossible. (Note that the paradox could easily be generated in the other direction so that Atalanta must first run half way, then half the remaining way, then half of that and so on, so that she must run the following endless sequence of fractions of the total distance: 1/2, then 1/4, then 1/8, then .)

A couple of common responses are not adequate. One mightas Simplicius ((a) On Aristotle's Physics, 1012.22) tells us Diogenes the Cynic did by silently standing and walkingpoint out that it is a matter of the most common experience that things in fact do move, and that we know very well that Atalanta would have no trouble reaching her bus stop. But this would not impress Zeno, who, as a paid up Parmenidean, held that many things are not as they appear: it may appear that Diogenes is walking or that Atalanta is running, but appearances can be deceptive and surely we have a logical proof that they are in fact not moving at all. Alternatively if one doesn't accept that Zeno has given a proof that motion is illusoryas we hopefully do notone then owes an account

*See more by searching '*Zeno's paradoxes*'.

of what is wrong with his argument: he has given reasons why motion is impossible, and so an adequate response must show why those reasons are not sufficient. And it won't do simply to point out that there are some ways of cutting up Atalanta's runinto just two halves, sayin which there is no problem. For if you accept all of the steps in Zeno's argument then you must accept his conclusion (assuming that he has reasoned in a logically deductive way): it's not enough to show an unproblematic division, you must also show why the given division is unproblematic.

Another responsegiven by Aristotle himselfis to point out that as we divide the distances run, we should also divide the total time taken: there is 1/2 the time for the final 1/2, a 1/4 of the time for the previous 1/4, an 1/8 of the time for the 1/8 of the run and so on. Thus each fractional distance has just the right fraction of the finite total time for Atalanta to complete it, and thus the distance can be completed in a finite time. Aristotle felt that this reply should satisfy Zeno, however he also realized (Physics, 263a15) that it could not be the end of the matter. For now we are saying that the time Atalanta takes to reach the bus stop is composed of an infinite number of finite pieces, 1/8, 1/4, and 1/2 of the total timeand isn't that an infinite time?

Of course, one could again claim that some infinite sums have finite totals, and in particular that the sum of these pieces is 1 the total time, which is of course finite (and again a complete solution would demand a rigorous account of infinite summation, like Cauchy's). However, Aristotle did not make such a move. Instead he drew a sharp distinction between what he termed a continuous line and a line divided into parts. Consider a simple division of a line into two: on the one hand there is the undivided line, and on the other the line with a mid-point selected as the boundary of the two halves. Aristotle claims that these are two distinct things: and that the latter is only potentially derivable from the former. Next, Aristotle takes the common-sense view that time is like a geometric line, and considers the time it takes to complete the run. We can again distinguish the two cases: there is the continuous interval from start to finish, and there is the interval divided into Zeno's infinity of half-runs. The former is potentially infinite in the sense that it could be divided into the latter actual infinity. Here's the crucial step: Aristotle thinks that since these intervals are geometrically distinct they must be physically distinct. But how could that be? He claims that the runner must do something at the end of each half-run to make it distinct from the next: she must stop, making the run itself discontinuous. (It's not clear why some other action wouldn't suffice to divide the interval.) Then Aristotle's full answer to the paradox is that the question of whether the infinite series of runs is possible or not is ambiguous: the potentially infinite series of halves in a continuous run is possible, while an actual infinity of discontinuous half runs is notZeno does identify an impossibility, but it does not describe the usual way of running down tracks!

It is hardfrom our modern perspective perhapsto see how this answer could be completely satisfactory. In the first place it assumes that a clear distinction can be drawn between potential and actual infinities, something that was never fully achieved. Second, suppose that Zeno's problem turns on the claim that infinite sums of finite quantities are invariably infinite. Then Aristotle's distinction will only help if he can explain why potentially infinite sums are in fact finite (and couldn't I potentially add 1 + 1 + 1 + , which does not have a finite total); or if he can give a reason why potentially infinite sums just don't exist. Or perhaps Aristotle did not see infinite sums as the problem, but rather whether completing an infinity of finite actions is metaphysically and conceptually and physically possible, an idea discussed at length in recent years: see Supertasks below. In this case we need an account of actions that makes precise the sense in which the continuous run is indeed a single action (using rest to individuate motions seems problematic, for humans are probably never completely still, and yet we perform distinct motionsbreathing, eating, skipping and so on). Finally, the distinction between potential and actual infinities has played no role in mathematics since Cantor tamed the transfinite numberscertainly the potential infinite has played no role in the modern mathematical solutions discussed here.